# By the way… Grades

- Total homework score: 50% of total grade
- Total lab score: 20% of total grade
- Exam: 30% of total grade

# Friday

- Beta mismatch invariant
- Chromaticity -- part II
    - due to sextupole field errors
    - correction, using sextupole magnets
    - effects of sextupole fields on transverse motion
- Magnet Edge effects
- Discuss (briefly) homework problems

# Mismatch Invariant

- Consider two solutions to $\beta'' + 4K\beta = const.$ through a focusing system
  - for example, one may be the periodic solution, the other a perturbed solution
- Then,

propagate original solution

propagate perturbed solution

$$
\begin{aligned}
J_{02} &= M J_{01} M^{-1} \\
J_{02} + \Delta J_2 &= M(J_{01} + \Delta J_1) M^{-1} \\
\Delta J_2 &= M \Delta J_1 M^{-1} \\
\det \Delta J_2 &= \det M \det \Delta J_1 \det M^{-1} \\
\det \Delta J_2 &= \det \Delta J_1
\end{aligned}
$$

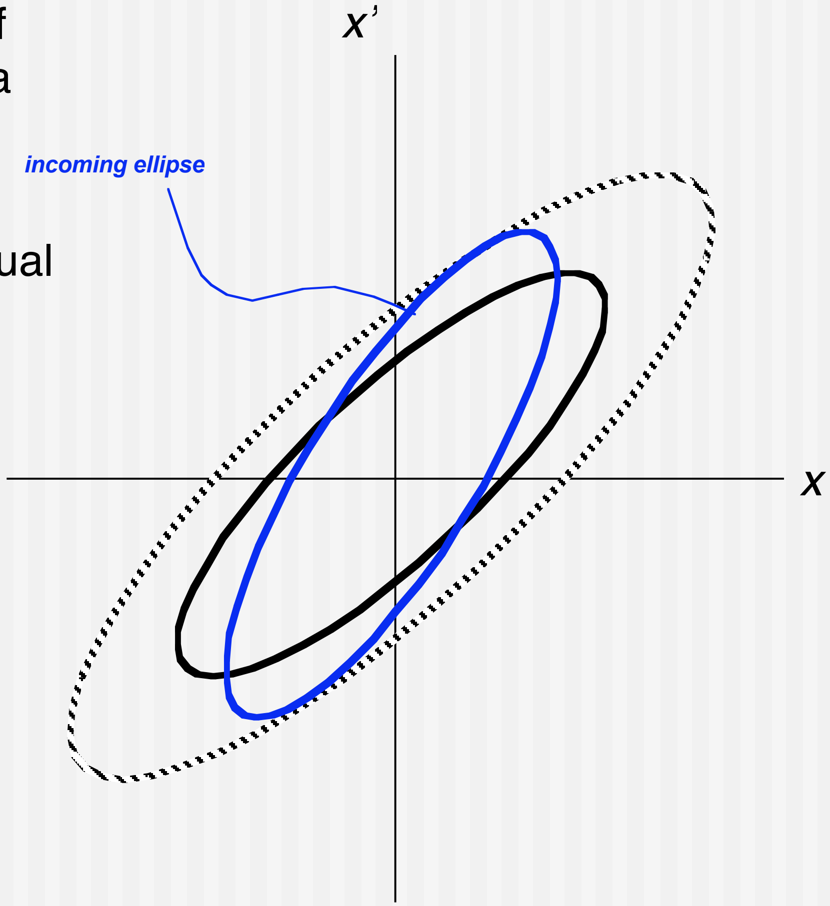
Thus, $\det \Delta J$ for two solutions is a constant along a beamline

# Expressions for Determinant of ΔJ

$$
\begin{aligned}
\det \Delta J &= \det(J_1 - J_0) \\
&= \begin{vmatrix} \Delta\alpha & \Delta\beta \\ -\Delta\gamma & -\Delta\alpha \end{vmatrix} \\
&= -\Delta\alpha^2 + \Delta\beta\Delta\gamma \\
&= 2 - (\beta_0\gamma_1 + \beta_1\gamma_0 - 2\alpha_0\alpha_1) \\
&= -\frac{\left(\frac{\Delta\beta}{\beta_0}\right)^2 + \left(\Delta\alpha - \alpha_0\frac{\Delta\beta}{\beta_0}\right)^2}{1 + \frac{\Delta\beta}{\beta_0}} < 0
\end{aligned}
$$

# Injection Mismatch and Emittance Dilution

- Suppose beam arrives through a transfer line into a synchrotron, but the beta function of the line is not matched to the periodic beta function of the ring...
- Particles will begin to follow phase space trajectories dictated by the ring lattice; actual nonlinearities of the real accelerator will cause their motion to decohere
- Net result: emittance dilution




if $\epsilon \sim \langle x^2 \rangle$, then

$$\boxed{\epsilon/\epsilon_0 = 1 - \frac{1}{2} \det \Delta J}$$

# Chromaticity -- Part II
# Sextupole Fields

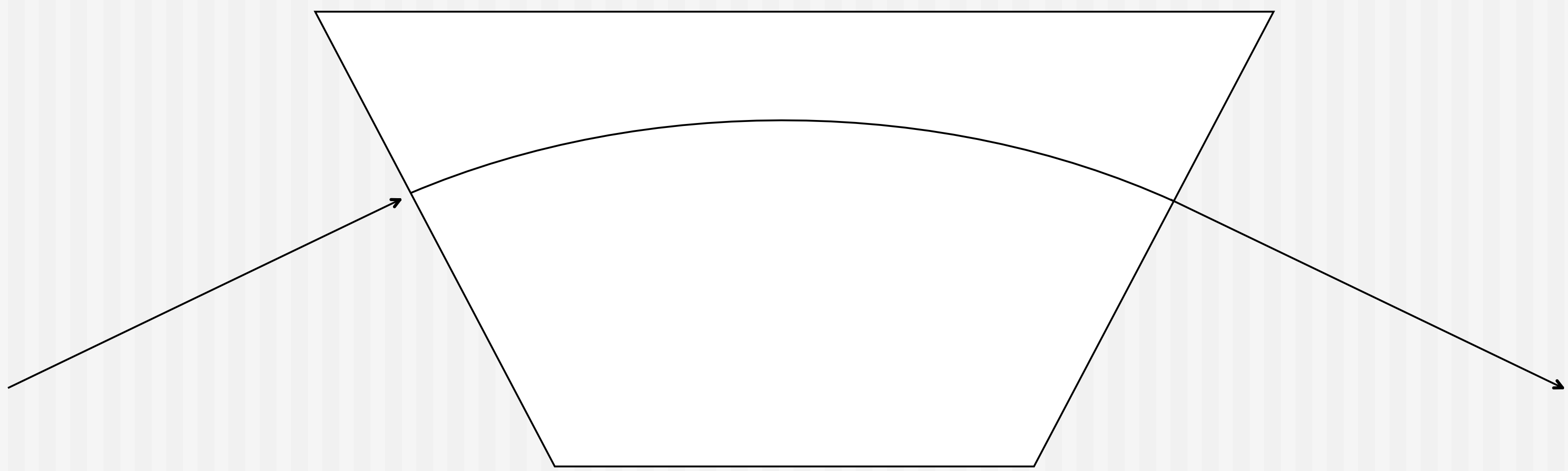
- Chromaticity due to sextupole field errors
  - $B_y = B_0 \, ( 1 + b_1 x + b_2 x^2 + \ldots)$
- Chromaticity correction, using sextupole magnets
  - $B_y = B'' \, ( x^2 - y^2)$
  - $B_x = 2 \, B'' \, xy$
- effects of sextupole fields on transverse motion

# Magnet Edges

- Need to look at effects at entrance and exit of magnets (bending magnets in particular).
- Will look at small angle/displacement approximations, as usual; more detailed descriptions can be found in various references (Wiedemann's book, for example)
- More important in lower energy and/or magnets which produce large bending angles

# Sector Magnets

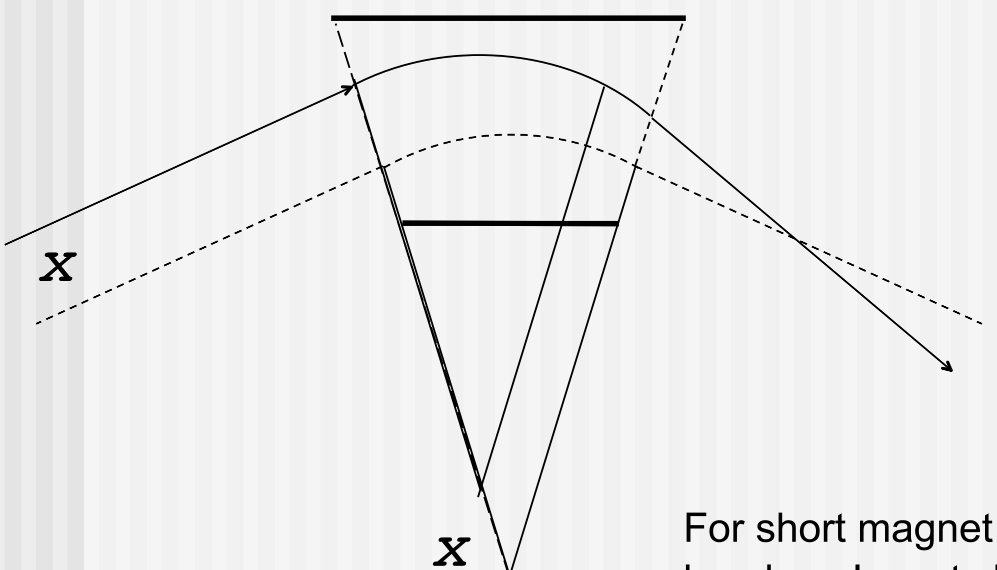
- Sector Dipole Magnet: “edge” of magnetic field is perpendicular to incoming/outgoing design trajectory:



Field points “*out of the page*”

# Sector Magnets & Sector Focusing

- Incoming ray displaced from ideal trajectory will experience more/less bending field, thus is "focused" toward axis in the *bend plane*:




$$\text{Extra path length} = ds = d\theta\ x$$
$$\text{so extra bend angle} = dx' = -ds/\rho$$
$$dx' = -(d\theta/\rho)x = -(1/\rho^2)x\ ds$$
$$\text{or, } x'' = -(1/\rho^2)x$$

$$\text{Thus, } K_x = 1/\rho^2,\ K_y = 0.$$

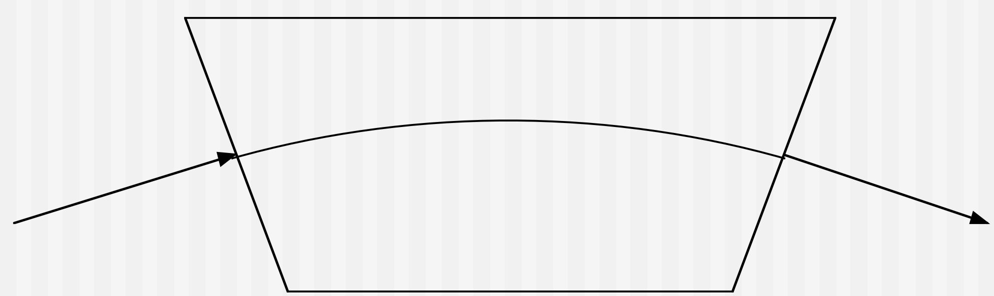
(as seen previously, with $B'$ = 0)

For short magnet with small bend angle, acts like lens in the bend plane with

$$\frac{1}{f_x} = \frac{\theta}{\rho}$$

# Edge Focusing

- In an ideal *sector magnet*, the magnetic field begins/ends exactly at $s = 0, L$ independent of transverse coordinates $x,y$ relative to the design trajectory.
  - *i.e.*, the face of the magnet is perpendicular to the design trajectory at entrance/exit



# Edge Focusing

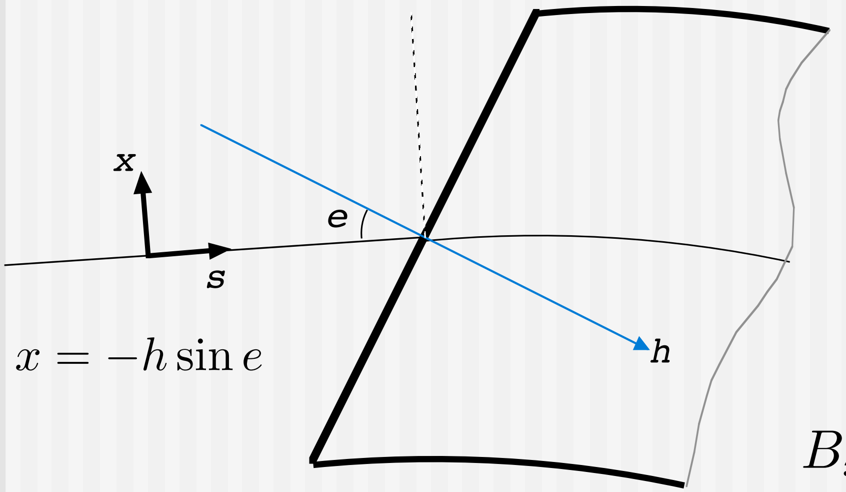
- However, could (and often do) have the faces at angles *w.r.t.* the design trajectory -- provides “edge focusing”


- Since our transverse coordinate *x* is everywhere perpendicular to *s*, then a particle entering with an offset will see more/less bending at the interface...

# So, How to Model Edges?

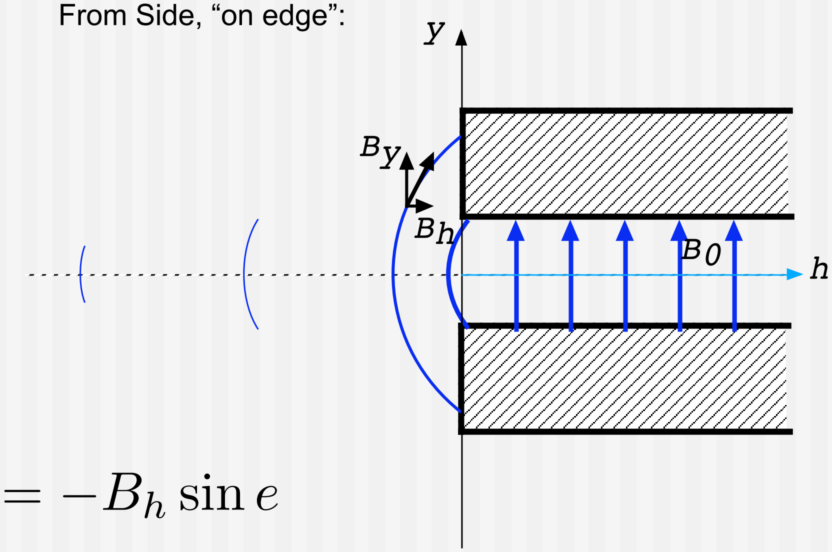
- In many cases, can consider edge effects to be perturbations to main motion, and treat as "impulse" kicks -- a "hard edge model"  (can do better modeling, if required...)

From Above:




$$x = -h \sin e$$

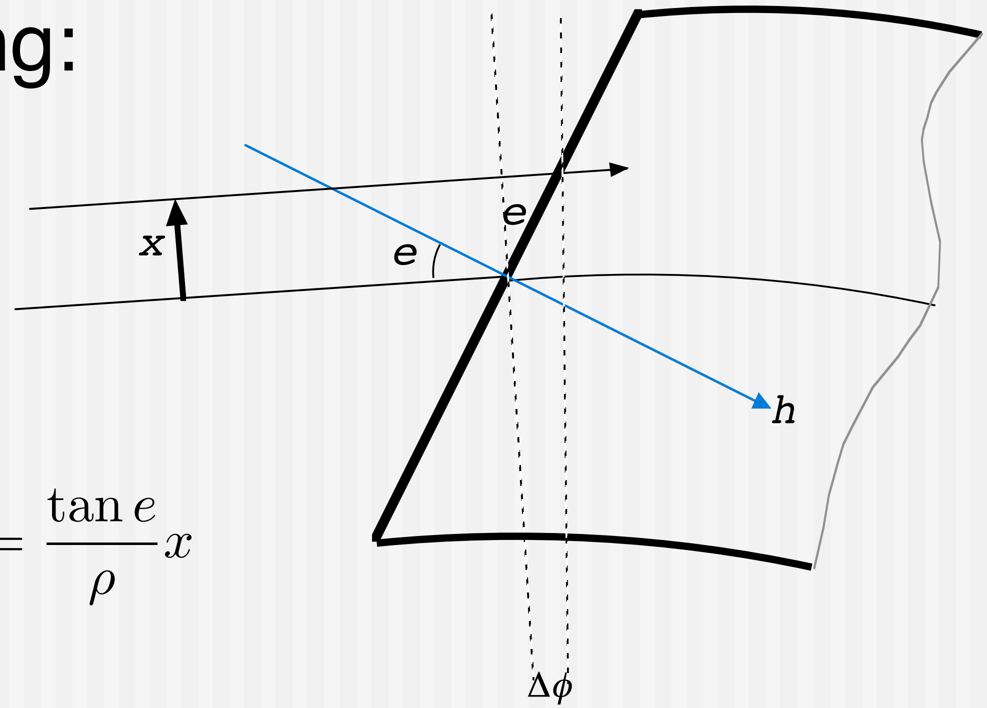
From Side, "on edge":




$$B_x = -B_h \sin e$$

# Edge Focusing -- radial

- Radial Defocusing:




$$\Delta x' = \Delta\phi = \frac{\Delta s}{\rho} = \frac{x\tan e}{\rho} = \frac{\tan e}{\rho}x$$

- So, for positive *x*, design trajectory “curves away” before particle reaches edge of magnetic field; thus, “defocusing” effect
- Similarly, upon exit

# Vertical Focusing at Edge

- From Maxwell's Eqs.,

$$\nabla \times \mathbf{B} = 0 \qquad \rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

  - and so... $$\Delta y' = -\frac{\tan e}{\rho} y$$

- If still not a believer, then ...

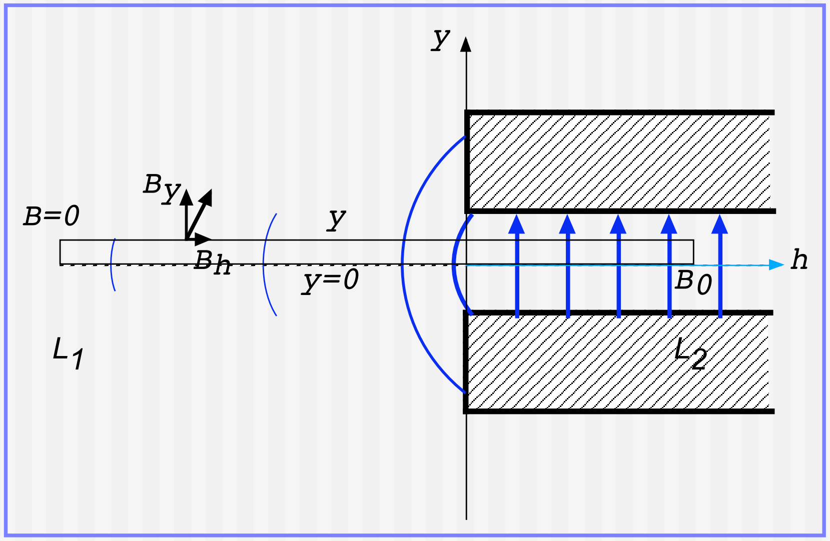
# Edge Focusing -- vertical

- Vertical Focusing:

$$\begin{aligned}\Delta y' &= \frac{\Delta p_y}{p} = \frac{ev \int B_x(y) ds}{pv} = \frac{1}{B\rho}\int B_x ds \\ &= -\frac{\sin e}{B\rho}\int B_h ds = -\frac{\tan e}{B\rho}\int (B_h \cos e) ds \\ &= -\frac{\tan e}{B\rho}\int_{L_1}^{L_2} \vec{B}\cdot\vec{ds}\end{aligned}$$

$$B_x = -B_h \sin e$$

$$B_x(y=0) = 0$$

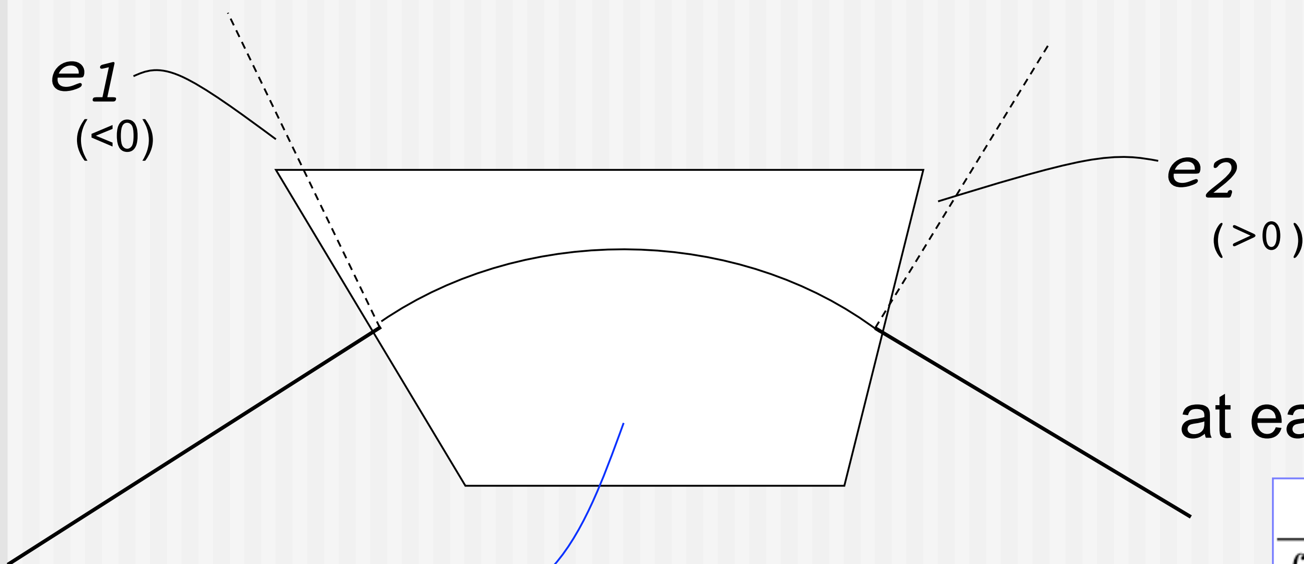



$$\begin{aligned}\oint \vec{B}\cdot\vec{ds} &= 0 + \int_{L_1}^{L_2} \vec{B}\cdot\vec{ds} - B_0 \cdot y + 0 = 0 \\ &\Downarrow \\ \int_{L_1}^{L_2} \vec{B}\cdot\vec{ds} &= yB_0\end{aligned}$$

$$\Longrightarrow \Delta y' = -\left(\frac{\tan e}{B\rho}\right) B_0 y = -\frac{\tan e}{\rho} y$$

# Total Bend Magnet:
# Sector + Edges

- Treat arbitrary edge angles as separate “lenses” at each end of a sector magnet…




inside, $K_x = 1/\rho^2$, $K_y = 0$

at each edge:

$$\boxed{\frac{1}{f_{x,y}} \approx \pm\frac{\tan e}{\rho}}$$

$$M_{mag} = M_{e_2} M_{sect} M_{e_1}$$

# Rectangular Bending Magnet

- "Rectangular" Dipole Magnet:

In bending plane, each edge acts as a lens with focal length:

$$\frac{1}{f} = -\frac{\theta/2}{\rho} = -\frac{\theta}{2\rho}$$

For Sector Magnet, then

hor: $$\frac{1}{f_x} \approx \frac{\theta}{\rho}$$

ver: $$\frac{1}{f_y} \approx 0$$

For Rectangular Magnet, then

hor: $$\frac{1}{f_x} \approx -\frac{\theta}{2\rho} + \frac{\theta}{\rho} - \frac{\theta}{2\rho} = 0$$

ver: $$\frac{1}{f_y} \approx \frac{\theta}{2\rho} + \frac{\theta}{2\rho} = \frac{\theta}{\rho}$$

# Tune change due to edge effects

- Suppose dipoles are located between quadrupoles of a FODO system, as in a large synchrotron
  - If use sector magnets:
    - $\Delta\nu_x = 1/4\pi \; \langle\beta\rangle \; \theta/\rho$ * no. of dipoles $= \langle\beta\rangle/(2\rho)$
    - $\Delta\nu_y = 0$
  - If use rectangular magnets:
    - $\Delta\nu_x = 0$
    - $\Delta\nu_y = 1/4\pi \; \langle\beta\rangle \; \theta/\rho$ * no. of dipoles $= \langle\beta\rangle/(2\rho)$

# Past Homework

- Problem Set 3 -- #1
- Problem Set 3 -- #4

# Homework Due Monday

- -- None --